VILNIUS UNIVERSITY
FACULTY OF PHYSICS
STUDIES OF LASERS LABORATORY

Laboratory work **No. KE – 6**

Research of laser Gaussian beams

Methodical instructions

Attention! Laser radiation sources are used during work – it is necessary to familiarize and strictly follow the respective rules of safety

Vilnius 2018

Purpose of the experiment

Investigate transformation of the laser Gaussian beam during propagation in free space and when passing focusing optical elements. Measure parameters of the Gaussian beam.

Experiment tasks

1. Align propagation of the He-Ne laser beam along the optical rail.
2. Investigate propagation of the He-Ne laser beam in free space.
3. Measure M^2 parameter of the He-Ne laser beam.
4. Align propagation of the laser diode module beam along the optical rail.
5. Investigate propagation of the laser diode module beam in free space.
6. Measure M^2 parameter of the laser diode module beam.

Theoretical topics

1. Laser Gaussian beams.
2. Parameters which characterize laser Gaussian beams.
3. Near field and far field zones.
4. Complex Gaussian beam parameters.
5. Gaussian beam divergence.
6. Transformation of the Gaussian beam when passing a lens.

Equipment and materials

1. He-Ne laser Thorlabs HNL020L, 2 mW, $\lambda = 632$ nm.
2. A laser diode module, $\lambda = 650$ nm.
3. A 1/1.8“ CCD camera Spiricon SP620U (matrix size: 7.1 mm x 5.4 mm, pixel size: 4.4 µm x 4.4 µm).
4. Aluminium mirrors.
5. A lens, $f = 20$ cm
6. Optical rail and mounts.
7. A measuring tape.

METHODICAL INSTRUCTIONS

The ideal Gaussian beam complex amplitude when propagating along the z direction is described as follows:

$$U(r,z) = A_0 \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w^2(z)}\right) \times \exp\left(-ikz - ik\frac{r^2}{2R(z)} + i\arctan\left(\frac{z}{z_R}\right)\right), \tag{1}$$

where A_0 is a amplitude, w_0 – the beam waist radius, $k = 2\pi/\lambda$ – the wavenumber, z_R – the Rayleigh length – distance at which $w(z_R) = \sqrt{2}w_0$, the Gaussian beam radius $w(z)$ along the z direction changes as:

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \tag{2}$$

the wavefront radius changes as:

$$R(z) = z\left[1 + \left(\frac{z_R}{z}\right)^2\right], \tag{3}$$

where the Rayleigh length is equal to:

$$z_R = \frac{\pi w_0^2}{\lambda}, \tag{4}$$

the divergence angle far from the beam waist is:

$$\theta = \frac{\lambda}{\pi w_0}. \tag{5}$$

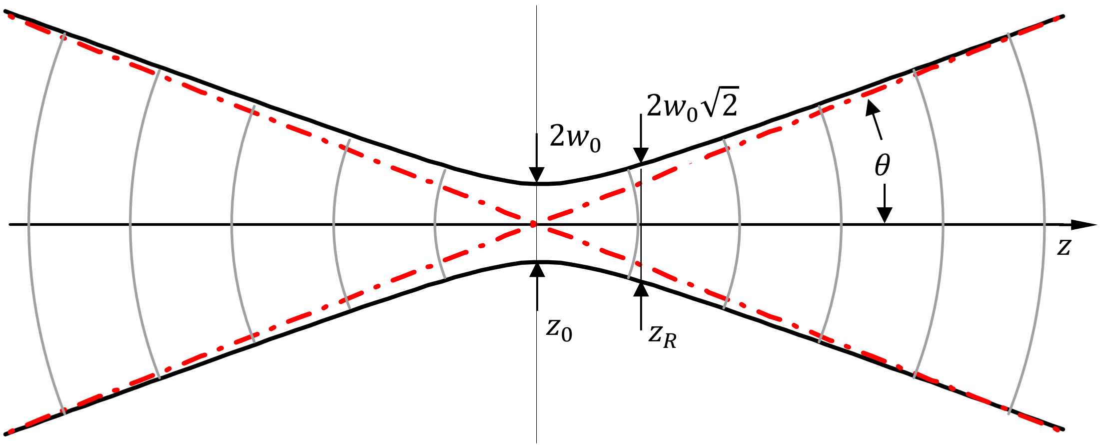


Fig. 1: Optical Gaussian beam.

Experimentally registered field intensity $I(r,z) = |U(r,z)|^2$ is:

$$I(r,z) = I_0\left(\frac{w_0}{w(z)}\right)^2 \exp\left(-\frac{2r^2}{w^2(z)}\right). \tag{6}$$

The beam radius is usually measured at $1/e^2 \approx 13.5\%$ intensity level. Another common beam diameter measurement is at the half intensity level (FWHM – full width at half maximum) which is 1.18 times less than the Gaussian beam radius at $1/e^2$ intensity level.

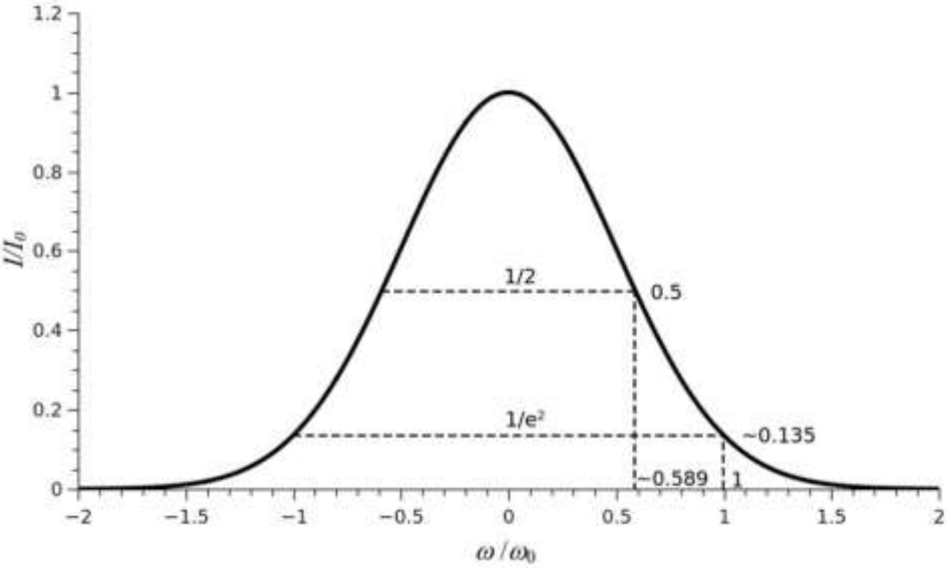


Fig. 2: Gaussian beam intensity distribution.

Real laser beams are not the ideal Gaussian beams. To estimate the deviations the real beam from the ideal Gaussian beam, a non-dimensional beam quality parameter M^2 has been introduced. It is a ratio of the real beam radius in waist (w_{0R}) and the divergence angle (θ_{0R}) to the ideal Gaussian beam radius in waist (w_0) and divergence angle (θ_0):

$$M^2 = \frac{w_{0R}\theta_{0R}}{w_0\theta_0}, \tag{7}$$

For the ideal Gaussian beam $M^2 = 1$. He-Ne laser generating TEM$_{00}$ transverse mode $M^2 < 1.1$. For multimode lasers which generate high power beams parameters M^2 can be 10 or more.

For the real optical beams:

$$w_{0R}\theta_{0R} = \frac{M^2\lambda}{\pi} > \frac{\lambda}{\pi}, \tag{8}$$

the beam radius for real beams at $1/e^2$ intensity level at *z* position from the beam waist is:

$$w_R(z) = w_{0R}\sqrt{1 + \left(\frac{z\lambda M^2}{\pi w_{0R}^2}\right)^2}, \tag{9}$$

the wavefront radius changes accordingly:

$$R_R(z) = z\left[1 + \left(\frac{\pi w_{0R}^2}{z\lambda M^2}\right)^2\right]. \tag{10}$$

The Rayleigh length is:

$$z_R = \frac{\pi w_{0R}^2}{M^2\lambda}, \tag{11}$$

According to ISO 11146 standard, the beam propagation parameter M^2 is described as follows:

$$M^2 = \frac{\pi}{\lambda} \frac{d_{\sigma 0} \theta_\sigma}{4}, \tag{12}$$

where λ is the wavelength, $d_{\sigma 0}$ – the beam waist diameter, θ_σ – the full beam divergence angle.

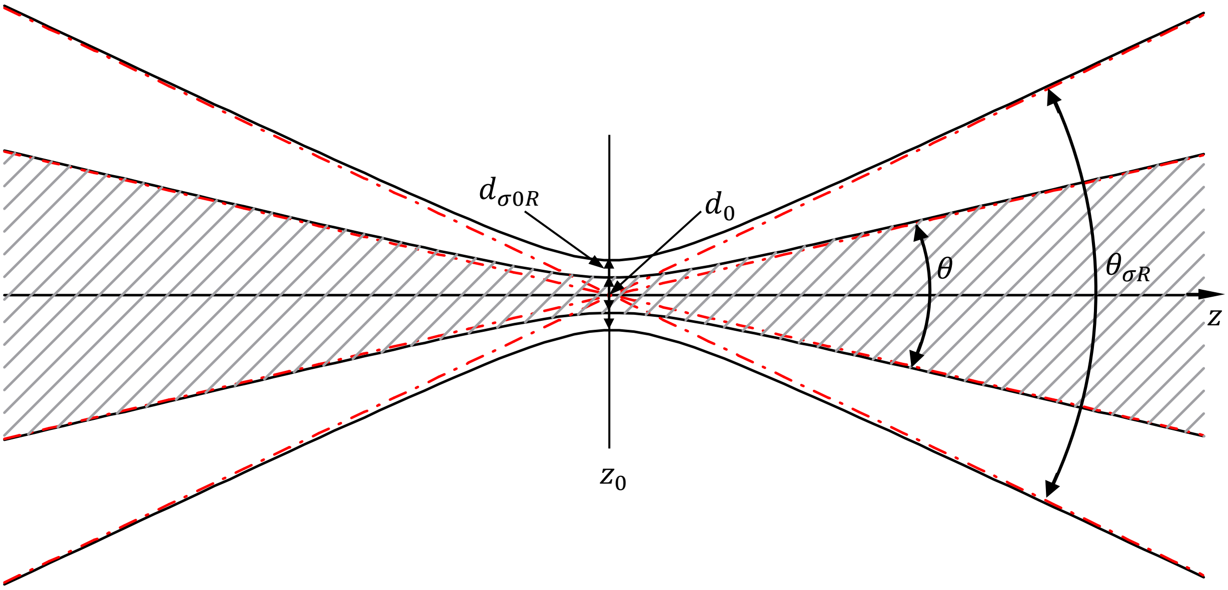


Fig. 3: Propagation of real beam.

The beam diameter at z position is calculated using the square root of second momentum of laser beam power density:

$$d_{\sigma x,y}(z) = 4\sigma_{x,y}(z), \tag{13}$$

Beam power density second momentum $\sigma_{x,y}^2(z)$ is described as follows:

$$\sigma_x^2(z) = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x-\bar{x})^2 E(x,y,z) \mathrm{d}x\mathrm{d}y}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x,y,z) \mathrm{d}x\mathrm{d}y}, \tag{14a}$$

$$\sigma_y^2(z) = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (y-\bar{y})^2 E(x,y,z) \mathrm{d}x\mathrm{d}y}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x,y,z) \mathrm{d}x\mathrm{d}y}, \tag{14b}$$

where $(x - \bar{x})$ and $(y - \bar{y})$ are distances from the beam centroid $(\bar{x}, \bar{y})$, $E(x, y, z)$ – the beam power density distribution. The position of the centroid is calculated as:

$$\bar{x} = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x E(x,y,z) \mathrm{d}x\mathrm{d}y}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x,y,z) \mathrm{d}x\mathrm{d}y}, \tag{15a}$$

and

$$\bar{y} = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} y E(x,y,z) \mathrm{d}x\mathrm{d}y}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x,y,z) \mathrm{d}x\mathrm{d}y}, \tag{15b}$$

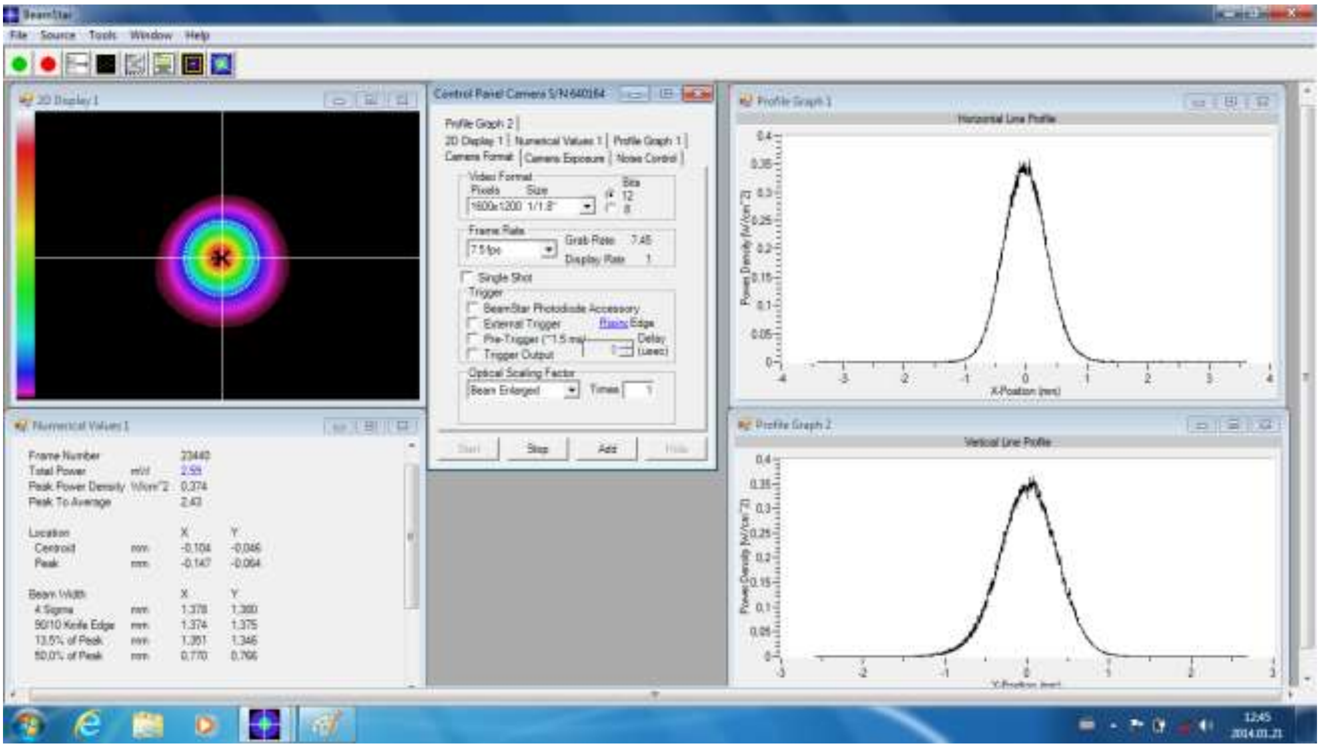
The beam divergence angle θ_σ is the angle of the cone which represents the laser beam radius or the diameter increasing when it propagated further away from the waist:

$$\theta_\sigma = \frac{d_{\sigma f}}{f}, \tag{16}$$

where $d_{\sigma f}$ is the diameter of the laser beam measured at the focal position of the focusing element’s rear plane, *f* – the focal length of the focusing element.

BEAMSTAR SOFTWARE METHODICAL INSTRUCTIONS

The software is launched by double clicking the icon on the desktop. If the camera is connected, the view depicted in Fig. 4 should be visible when the software is launched.



```
Frame Number                 23440
Total Power          mW      2.59
Peak Power Density   W/cm^2  0.374
Peak To Average              2.43

Location                     X        Y
  Centroid           mm      -0.104   -0.046
  Peak               mm      -0.147   -0.064

Beam Width                   X        Y
  4 Sigma            mm      1.378    1.380
  90/10 Knife Edge   mm      1.374    1.375
  13.5% of Peak      mm      1.351    1.346
  50.0% of Peak      mm      0.770    0.766
```

Video Format: Pixels Size 1600x1200 1/1.8"; Bits ● 12 ○ 8
Frame Rate: 7.5 fps; Grab Rate 7.45; Display Rate 1
☐ Single Shot
Trigger:
☐ BeamStar Photodiode Accessory
☐ External Trigger Rising Edge
☐ Pre-Trigger (~1.5 ms) Delay (usec)
☐ Trigger Output
Optical Scaling Factor: Beam Enlarged; Times 1

Fig. 4: Main window of BeamStar software.

In the main window the **Control Panel**, **2D Display 1**, profile (**Profile Graph 1** and **Profile Graph 2**) display windows should be visible together with **Numerical Values 1** window. In addition, **1D Gaussian Fit**, **1D Top Hat Fit** and other windows can also be opened. The number of the displayed windows can be changed as the user wishes. Additional windows can be added by clicking **Add** in the **Control Panel**. Tabs opened in the **Control Panel** window are used to control camera and the opened windows. When any of the windows is closed, the corresponding tab closes as well. The measurement is started by clicking **Start** in the control panel window or ● in the main window and be stopped by clicking **Stop** or ● in the same windows.

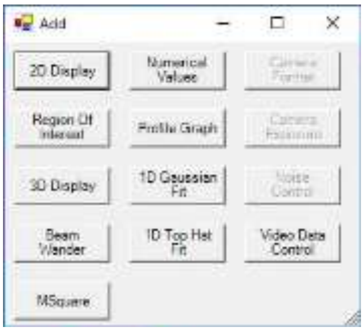
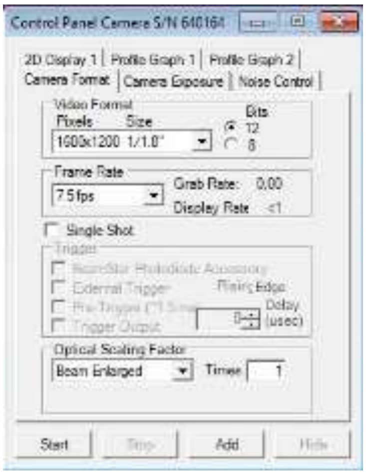


Fig. 5: Window where additional display windows can be selected.

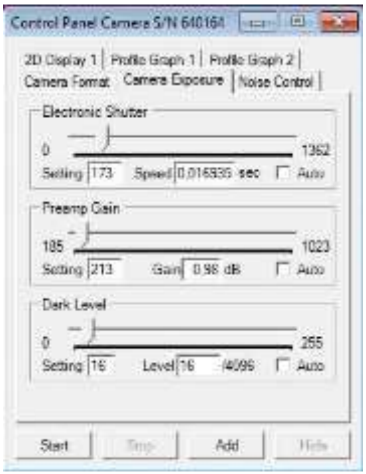
The **Video Format**, dynamic range (**Bits)**, **Frame rate** and **Trigger** options are set in the



Video Format
Pixels Size: 1600x1200 1/1.8"
Bits: ● 12 ○ 8
Frame Rate: 7.5 fps
Grab Rate: 0.00
Display Rate <1
☐ Single Shot
Trigger
☐ External Trigger
Rising Edge
☐ Trigger Output
Delay (usec): 0
Optical Scaling Factor: Beam Enlarged
Times: 1

Fig. 6: Camera format settings tab.

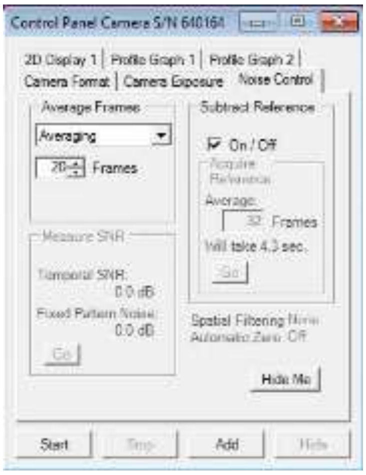
Camera Format tab. When a beam expander or objective is used, the **Optical Scaling Factor** is set not equal to 1. In this case (no expander or objective) it needs to be set to 1. The video format must be selected to be **1600x1200 1/1.8“**, dynamic range – **12** bit and appropriate frame rate.



Electronic Shutter
0 – 1362
Setting: 173
Speed: 0.016935 sec
☐ Auto
Preamp Gain
185 – 1023
Setting: 213
Gain: 0.98 dB
☐ Auto
Dark Level
0 – 255
Setting: 16
Level: 16 /4096
☐ Auto

Fig. 7: Camera temporal settings tab.

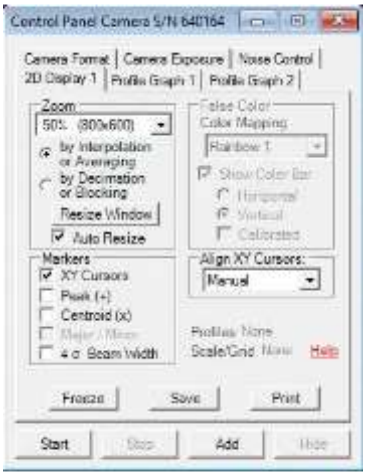
In the **Camera Exposure** tab exposure (**Electronic Shutter**), amplification (**Preamp Gain**) and dynamic background level (**Dark Level**) can be set. In this case select automatic exposure and background level setting (click **Auto**).



Average Frames: Averaging
20 Frames
Subtract Reference
☑ On / Off
Acquire Reference
Average: 32 Frames
Will take 4.3 sec.
Measure SNR
Temporal SNR: 0.0 dB
Fixed Pattern Noise: 0.0 dB
Spatial Filtering None
Automatic Zero: Off

Fig. 8: Signal/noise settings tab.

In the **Noise Control** tab averaging and reference subtraction can be set. In the **Average Frames** tab set 5 – 10 frame averaging.

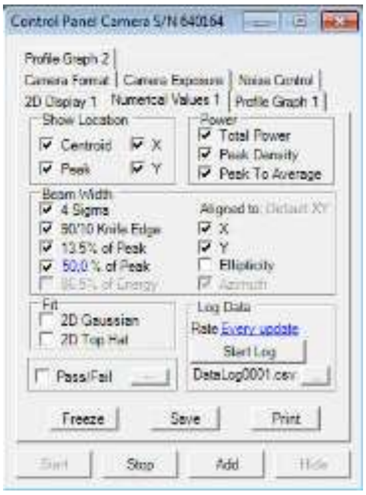


Zoom: 50% (800x600)
● by Interpolation or Averaging
○ by Decimation or Blocking
☑ Auto Resize
False Color
Color Mapping: Rainbow 1
☑ Show Color Bar
○ Horizontal
● Vertical
☐ Calibrated
Markers
☑ XY Cursors
☐ Peak (+)
☐ Centroid (x)
☐ Major / Minor
☐ 4 σ Beam Width
Align XY Cursors: Manual
Profiles: None
Scale/Grid: None

Fig. 9: Intensity profile display settings tab.

In the **2D display** window intensity distribution in the XY plane is depicted. In the **Control Panel** window’s **Zoom** tab it is also possible to set the depicted view resolution and size, select cursors (**XY cursors**). In the **Markers** tab one can select peak (**Peak(+)**) and centroid (**Centroid(×)**) marker display. In addition, the **Align XY Cursors** tab allows selecting the method to determine XY cursor position: **Manual**, mathematical peak estimation (**Peak**) or mathematical centroid position estimation (**Centroid**). Clicking **Freeze/Unfreeze** stops/launches view refreshing in **2D display** window. Clicking **Save** saves data displayed in the window using the selected format. In the **Numerical Values** window beam mathematical characterization results are displayed. The displayed quantities are selected in the **Control Panel** window tab. Clicking **Freeze/Unfreeze** stops/starts refreshing of **Numerical Values** window.

In **Profile Graph** windows intensity profiles are displayed and the display options are set in the corresponding **Control Panel** window tabs. Select **Line Profile** in **Profile Type.** In the axes tab **Axis** select one of the axes (**X** or **Y**). In **Align to** tab select the method for estimating profile position.



Show Location
☑ Centroid ☑ X
☑ Peak ☑ Y

Power
☑ Total Power
☑ Peak Density
☑ Peak To Average

Beam Width
☑ 4 Sigma
☑ 90/10 Knife Edge
☑ 13.5% of Peak
☑ 50.0 % of Peak
☐ 86.5% of Energy

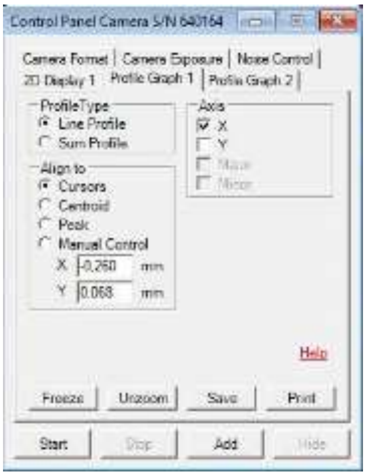
Aligned to: Default XY
☑ X
☑ Y
☐ Ellipticity
☑ Azimuth

Fit
☐ 2D Gaussian
☐ 2D Top Hat

☐ Pass/Fail

Log Data
Rate Every update
DataLog0001.csv

Fig. 10: Measurement results display settings tab.



ProfileType
● Line Profile
○ Sum Profile

Align to
● Cursors
○ Centroid
○ Peak
○ Manual Control
X -0.260 mm
Y 0.068 mm

Axis
☑ X
☐ Y
☐ Major
☐ Minor

Fig. 11: Intensity profile display settings tab.

Clicking **Freeze/Unfreeze** stops/starts refreshing data in the **Profile Graph** window. Clicking **Save** saves the displayed data using the selected format.

Before measurements set the coordinate axis directions along x and y. By Clicking or selecting from menu **Tools>Options** the **Axis Alignment Control** window opens where it is necessary to select **Default XY**.

☑ Align all Displays to:
● Default XY ○ Beam Axis

Fig. 12: Beam measurement axis settings window.

For more accurate selection of the camera position and accurate measurement of the distance to the lens and the distance between the lens and the CCD camera, it is necessary to take into account the positions of the front principal plane *H*, the rear principal plane *H‘* and the CCD camera position. The arrow on the lens marks propagation direction of the beam. The front principal plane *H* is 11.65 mm distance from the left edge of the optical mount. The rear principal plane *H‘* is within 11.65 mm distance from the right edge of the optical mount. The CCD matrix is 6 mm from the left edge of the CCD camera optical mount.

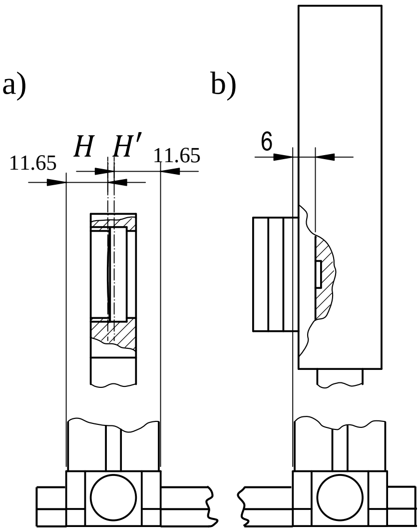


Fig. 13: a) The lens front (*H*) and rear (*H’*) principal plane position with respect to the edges of the lens optical mount. b) the CCD matrix position with respect to the edge of the camera optical mount.

Experiment procedure

1. Alignment of the He-Ne laser beam along the optical rail.

- The CCD camera must be used together with all 3 included filters. Connect the CCD camera to the computer and turn the computer on.
- Make sure that the laser power supply is connected to the electric socket and to the He-Ne laser head. Turn on He-Ne laser by turning the key in the power supply clockwise.
- Move the CCD camera (CCD) towards the left edge of the optical rail.
- Place and fasten with screws the alignment mirrors DV1 and DV2.

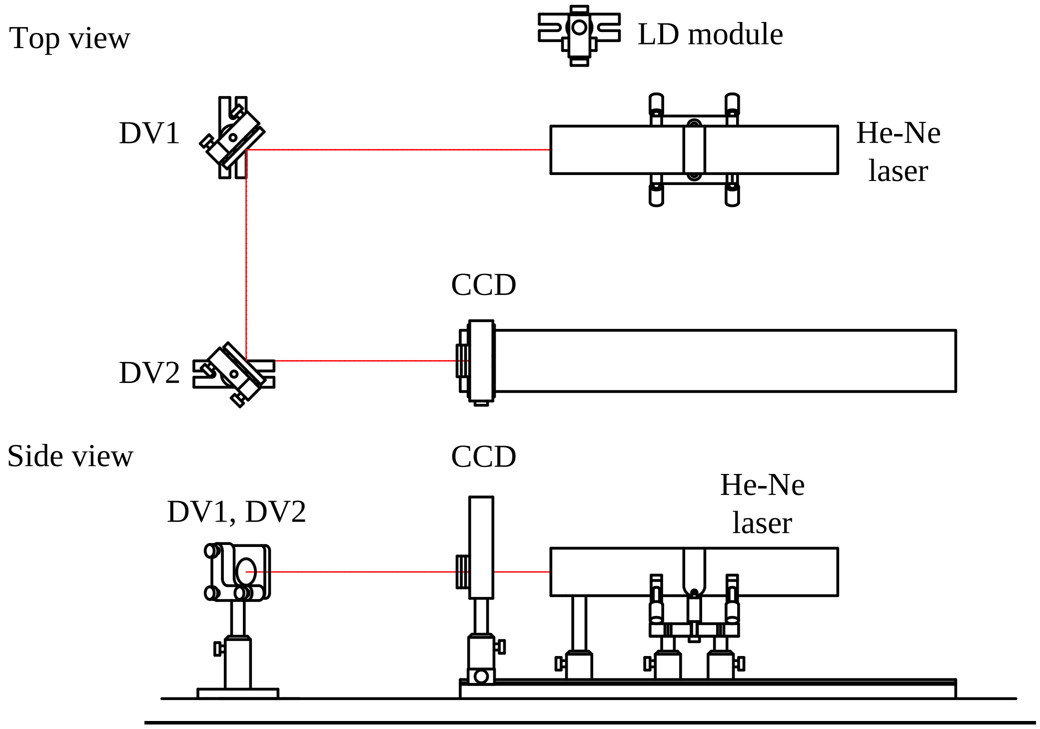


Fig. 14: Experimental setup. He-Ne laser, LD module – laser diode module, DV1 and DV2 – alignment mirrors, CCD – CCD camera.

- Using mirror DV1 align the laser beam to match the center of the CCD camera.
- Move the CCD camera towards the other end of the optical rail. Using mirror DV2 align the laser beam to match the center of the CCD camera.
- Move the CCD camera towards (0) position on the optical rail. Repeat alignment of mirror DV1.
- Repeat alignment procedures until the beam propagates along the optical rail.

2. Investigation of He-Ne laser beam propagation in free space.

- Move the CCD camera towards (0) position on the optical rail. The center of the laser beam must match the center of the CCD camera matrix.
- Measure the distance between the laser output coupler and the camera.

- Make sure that **Peak(+)** position matches or is close to the **Centroid(×)** position. If there is a large mismatch, cover the laser beam and click **Zero the Background** which subtracts the background. Alternatively, in the **Noise Control** tab measure the reference intensity distribution when laser beam is covered and subtract it.
- Record the beam intensity distribution and estimate the beam diameter $d_{13.5}$ at level $1/e^2$ (13.5%) and the beam diameter $d_\sigma = 4\sigma$ at X and Y axes.
- Measure the beam diameters $d_{13.5}$ $1/e^2$(13.5%) and $d_\sigma = 4\sigma$ at X and Y axes dependences on position *z*, when the CCD camera is moving along the optical rail towards its other end with 50 mm steps,
- Graphically depict $d_{13.5}(z)$, and $d_\sigma(z)$ dependence on *z*.
- From part of the graph $d(z) = f(z)$ which can be approximated linearly, determine beam divergence angle:

$$\theta_i = 2 \cdot \arctan\left(\frac{d_{i\,2}/2 - d_{i\,1}/2}{z_2 - z_1}\right), \qquad (17)$$

and waist position z_{0i} with reference to (0) position on the optical rail. Here *i* are indices of *x* or *y*.

3. Measurement of He-Ne laser beam M^2 parameter.

- Move the CCD camera towards the right end of the optical rail.
- Place a lens L at (0) position on the optical rail. Adjust the height of the lens optical mount so that the centre of the laser beam would match the center of the CCD matrix.

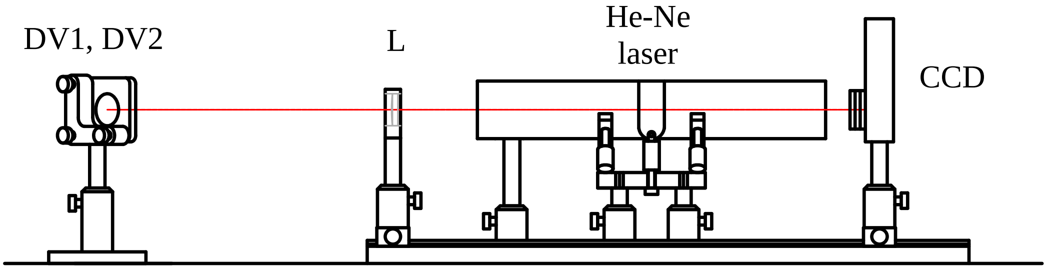


Fig. 15: Experimental setup for measurement of M^2 parameter. He-Ne laser, DV1 and DV2 – alignment mirrors, L – lens, CCD – CCD camera.

- While moving the CCD camera along the optical rail, determine position of beam waist z_0 (where $d_{\sigma 0} = 4\sigma_0$ along X and Y axes is minimal).
- While moving the CCD camera along the optical rail, determine position $z_{(-)}$ before beam waist and $z_{(+)}$ after beam waist where beam diameter is $d_{\sigma\mathrm{R}} = \sqrt{2} d_{\sigma 0}$ along X and Y axes.
- Measure laser beam diameters $d_{\sigma x} = 4\sigma_x$ and $d_{\sigma y} = 4\sigma_{xy}$ at 6 or more different

positions between $z_{(-)}$ and $z_{(+)}$. At least 3 measurements must be at positions before beam waist and at least 3 after beam waist.

- Measure laser beam diameters $d_{\sigma x} = 4\sigma_x$ and $d_{\sigma y} = 4\sigma_y$ at 3 or more positions before beam waist z_0 when distance is greater than $2\left|z_0 - z_{(-)}\right|$ from the beam waist.
- Measure laser beam diameters $d_{\sigma x} = 4\sigma_x$ and $d_{\sigma y} = 4\sigma_y$ at 3 or more positions after beam waist z_0 when distance is greater than $2\left|z_{(+)} - z_0\right|$from the beam waist.

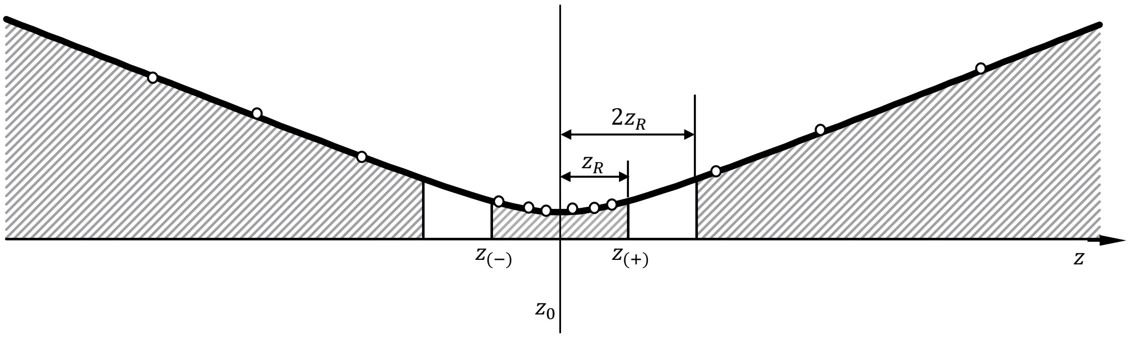


Fig. 16: Positions where beam diameter measurements should be performed.

- Approximate the measured beam diameter $d_{\sigma x}(z)$ and $d_{\sigma y}(z)$ dependence on position *z* using hyperbolic function:

$$d_{\sigma x}(z) = \sqrt{a_x + b_x z + c_x z^2}, \tag{18a}$$

and

$$d_{\sigma y}(z) = \sqrt{a_y + b_y z + c_y z^2}. \tag{18b}$$

Parameters a_x, b_x, c_x and a_y, b_y, c_y are calculated using digital curve approximation methods.

Graphically depict measured diameter dependence on position z and respective approximation curves.

- Using parameters a_x, b_x, c_x and a_y, b_y, c_y calculate beam propagation parameters:

$$z_{0i} = \frac{-b_i}{2c_i}, \tag{19}$$

$$d_{\sigma i0} = \frac{1}{2\sqrt{c_i}}\sqrt{4a_i c_i - b_i^2}, \tag{20}$$

$$\theta_{\sigma i} = \sqrt{c}, \tag{21}$$

$$z_{Ri} = \frac{1}{2c_i}\sqrt{4a_i c_i - b_i^2}, \tag{22}$$

$$M_i^2 = \frac{\pi}{8\lambda}\sqrt{4a_i c_i - b_i^2}, \tag{23}$$

where *i* are indices of *x* or *y*, λ – laser wavelength.

- Calculate astigmatic beam waist difference Δz_z:

$$\Delta z_z = \left|z_{0x} - z_{0y}\right|, \tag{24}$$

- Calculate laser beam waist position with respect to lens front principal surface *H* at X axis:

$$z_{0x}^{\text{before}} = V^2 x_2 + f, \tag{25}$$

where

$$x_2 = z_{0x}^{\text{behind}} - f, \tag{26}$$

and

$$V = \frac{f}{\sqrt{z_{Rx}^2 + x_2^2}}, \tag{27}$$

where f is focal length of the lens, z_{0x}^{behind} – laser beam waist position with respect to lens rear principal surface *H‘* which is calculate using formula (19), z_{Rx} – beam Rayleigh length after lens which is calculated using formula (22).

Calculate beam waist diameter before the lens:

$$d_{\sigma x0}^{\text{before}} = V d_{\sigma x0}^{\text{behind}}. \tag{28}$$

Calculate beam Rayleigh length before the lens:

$$z_{Rx}^{\text{before}} = V^2 z_{Rx}^{\text{behind}}. \tag{29}$$

Calculated beam divergence angle before the lens:

$$\theta_x^{\text{before}} = \frac{\theta_x^{\text{behind}}}{V}, \tag{30}$$

- Using formulae (25) – (30) calculate laser beam parameters before the lens along the Y axis.
- Compare beam waist positions and divergence angles calculated form formula (25) and (30) with measured values (task 2).

4. **Alignment of the laser diode module beam propagation along the optical rail.**

- Turn on the laser diode module.

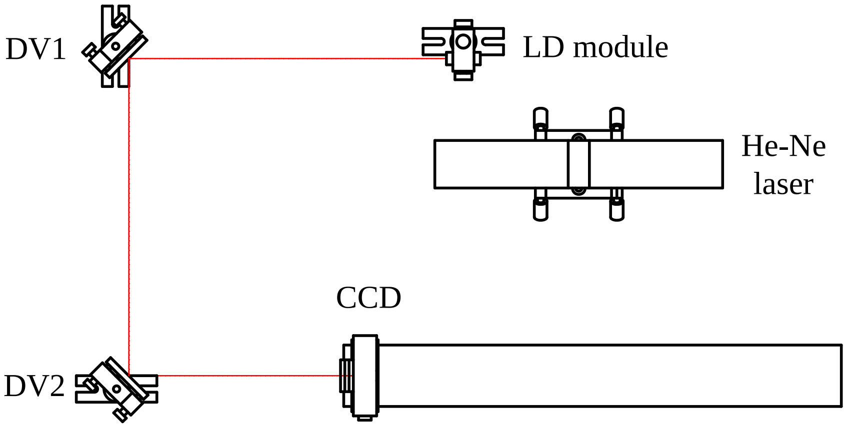


Fig. 17: Experimental setup. LD module – laser diode module, He-Ne laser, DV1 and DV2 – alignment mirrors, CCD – CCD camera.

- Replace and fasten with screws the alignment mirrors DV1 and DV2.
- Using instructions described in the task 1 align the experimental setup.

5. Investigation of laser diode module beam propagation in free space.

- Using instructions described in the task 2 measure the laser diode beam divergence angles θ_x and θ_y and the position of the beam waist.
- Compare the results with He-Ne laser beam parameters.

6. Measurement of laser diode module beam M^2 parameter.

- Using instructions described in the task 3 measure the laser diode beam propagation parameters.
- Compare the results with He-Ne laser beam parameters.

Literature

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